Chapter 11-1 Points, Lines, and Planes

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. [12]

1. Which of the following statements is false?
   (1) If two lines are parallel, then they must be coplanar.
   (2) Every triangle is a plane figure.
   (3) Three parallel lines must be coplanar.
   (4) Two perpendicular lines must be coplanar.

2. If two lines have exactly one point in common, how many planes contain those lines?
   (1) none      (3) two
   (2) one       (4) infinitely many

In 3–5, use the given figure.

3. Which of the following is a point in the plane determined by the points $H$, $B$, and $C$?
   (1) $A$
   (2) $D$
   (3) $F$
   (4) $G$

4. Which of the following is a point in the plane determined by the lines $\vec{HE}$ and $\vec{FE}$?
   (1) $A$
   (2) $B$
   (3) $C$
   (4) $D$

5. Which two lines are skew?
   (1) $\vec{AF}$ and $\vec{CD}$
   (2) $\vec{FE}$ and $\vec{GD}$
   (3) $\vec{FG}$ and $\vec{HC}$
   (4) $\vec{AH}$ and $\vec{CD}$

6. How many lines are parallel to a given line through a given point not on the line?
   (1) none      (3) two
   (2) one       (4) infinitely many

PART II

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [8]

7. Using the given cube:
   a. Name 4 lines perpendicular to $\vec{ED}$.
   b. Name 3 lines parallel to $\vec{ED}$.
   c. Name 4 lines skew to $\vec{ED}$.
8. For each of the following, answer none, one, or infinitely many.

   a. How many lines can pass through a point in a plane?

   b. How many lines can pass through a point in space?

   c. How many planes can contain a given line in space?

   d. How many lines can pass through two points in a plane?

   e. How many planes can pass through a pair of skew lines?

   f. How many planes contain a line and a point not on the line?
1. Through a point outside a given plane, how many planes can be drawn perpendicular to the given plane?
   (1) none
   (2) one
   (3) two
   (4) infinitely many

2. Which of the following statements is true?
   (1) A line perpendicular to a plane is perpendicular to every line in the plane.
   (2) The plane angles of a dihedral angle are never congruent.
   (3) Two planes perpendicular to the same line at a given point coincide.
   (4) Two planes perpendicular to the same plane intersect in exactly one line.

3. How many planes are determined by four points which do not all lie in the same plane?
   (1) one
   (2) two
   (3) three
   (4) four

4. Point \( P \) is on line \( l \) and line \( l \) is in plane \( q \). How many lines in plane \( q \) are perpendicular to \( l \) at \( P \)?
   (1) none
   (2) one
   (3) two
   (4) infinitely many

5. In the given figure, a plane angle of the dihedral angle is
   (1) \( \angle HFG \)
   (2) \( \angle CFA \)
   (3) \( \angle AFG \)
   (4) \( \angle BFG \)

6. How many lines in space are perpendicular to a given line at a point on the line?
   (1) none
   (2) one
   (3) two
   (4) infinitely many
PART II

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [8]

7. Identify the following statements as true or false.
   a. Two planes perpendicular to the same plane never have points in common.
      Answer: False
   b. If a line is perpendicular to a plane then every plane containing the line is perpendicular to the given plane.
      Answer: True
   c. If a line is perpendicular to each of two intersecting lines at their point of intersection, then the line is perpendicular to the plane determined by these lines.
      Answer: True
   d. If two planes are perpendicular to each other, then each line in one plane is perpendicular to a line in the other plane.
      Answer: False

8. In the given figure, plane \( m \) bisects the dihedral angle formed by planes \( n \) and \( l \). The measure of plane \( \angle GHS \) is \( 7x + 2 \) and the measure of plane \( \angle GHR \) is \( 3x + 10 \).
   a. Find \( x \).
   b. Find the measure of \( \angle GHR \) and \( \angle SHR \).
   c. Find the measures of the dihedral angle formed by planes \( n \) and \( l \) and the dihedral angle formed by planes \( m \) and \( n \).
1. Which of the following statements is true?
   (1) If a straight line is parallel to a plane, it is parallel to every line in the plane.
   (2) If two planes are parallel, any line in one plane is parallel to any line in the other.
   (3) Any two planes parallel to the same plane are perpendicular to each other.
   (4) If a line intersects one of two parallel planes, then it intersects the other.

2. If point $P$ lies outside of plane $N$, how many lines parallel to plane $N$ can be drawn through point $P$?
   (1) none
   (2) one
   (3) two
   (4) infinitely many

3. If point $P$ lies outside of plane $N$, how many planes parallel to plane $N$ can be drawn through point $P$?
   (1) none
   (2) one
   (3) two
   (4) infinitely many

4. If line $\vec{AB}$ is perpendicular to a plane, how many planes containing $\vec{AB}$ can be drawn parallel to the plane?
   (1) none
   (2) one
   (3) two
   (4) infinitely many

5. If two parallel planes are intersected by a third plane, how many lines of intersection are there?
   (1) one
   (2) two
   (3) three
   (4) infinitely many

6. Which of the following statements is false?
   (1) A line parallel to the intersection of two planes is parallel to each of the planes.
   (2) Two planes parallel to the same line are parallel to each other.
   (3) The shortest segment from a point to a plane is the perpendicular from that point.
   (4) Parallel segments drawn to a plane from points in a line parallel to the plane are congruent.
7. For a–d, fill in the blank with the number of the phrase that makes each statement true. There may be more than one answer. Select from the following list:

<table>
<thead>
<tr>
<th>(1) sometimes parallel</th>
<th>(3) always parallel</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2) sometimes perpendicular</td>
<td>(4) always perpendicular</td>
</tr>
</tbody>
</table>

a. Two planes perpendicular to the same plane are __________ and __________ to each other.
b. A plane perpendicular to a line that is parallel to a given plane is __________ to that plane.
c. If three planes intersect each other in two and only two lines, then two of the planes are __________ and __________.
d. Two planes perpendicular to the same line are __________ to each other.

8. In the given figure:

a. Name the lines parallel to \( \overrightarrow{BC} \).

b. Name the planes parallel to \( \overrightarrow{CF} \).

c. Name the lines perpendicular to \( \overrightarrow{CD} \).

d. Name the planes perpendicular to \( \overrightarrow{FE} \).
Chapter 11-4 Surface Area of a Prism

PART I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. [12]

1. What is the total surface area of a rectangular prism whose dimensions are x by x by 3x?
   (1) 3x^2 sq units
   (2) 4x^2 sq units
   (3) 10x sq units
   (4) 14x^2 sq units

2. What is the total surface area of a rectangular solid that is 4 inches by 7 inches by 10 inches?
   (1) 42 sq units
   (2) 138 sq units
   (3) 276 sq units
   (4) 280 sq units

3. What is the formula for the total surface area of a rectangular solid whose dimensions are a by b by c?
   (1) abc
   (2) 2a + 2b + 2c
   (3) 2abc
   (4) 2ab + 2ac + 2bc

4. As indicated in the given rectangular solid, the area of two of the faces are 35 and 56 square units.

   ![Rectangular Solid]

   If the sides of the solid have integer lengths, what is the total length of all the edges of the solid?
   (1) 40
   (2) 60
   (3) 80
   (4) 280

5. If the length of an edge of a cube is 6.25 inches, what is the total surface area of the cube to the nearest tenth of an inch?
   (1) 37.5 in.^2
   (2) 156.3 in.^2
   (3) 234.4 in.^2
   (4) 244.1 in.^2

6. The lateral edge of a prism is an altitude. This solid is what type of prism?
   (1) right prism
   (2) rectangular solid
   (3) parallelepiped
   (4) cube
PART II

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [8]

7. a. What is the total number of right angles formed by the edges of any rectangular prism?
   
   Answer: 24

   b. What is the least number of faces that a polyhedron can have?
   
   Answer: 4

   c. What is the least number of vertices that a polyhedron can have?
   
   Answer: 4

   d. What is the least number of edges that a polyhedron can have?
   
   Answer: 6

8. The base of a right prism 30 inches high is a rhombus with diagonals of length 8 and 6. Find the total surface area of the prism.

   Solution:
   
   The diagonals of the rhombus form four right triangles with legs of length 3 and 4 and hypotenuses of length 5. Thus, the length of each side of the base is 5.

   \[ S = 5 \times 648 \text{ in.}^2 \]

   \[ S = 5 \times 48 \times 1600 \]

   \[ S = 2 \times 4 \times 3 \times 4 \]

   \[ T = 5 \times (30 + 4) \]

   \[ T = 1350 \]

   \[ B = 5 \times 648 \]

   \[ B = 3240 \]

   \[ L = 5 \times 12 \]

   \[ L = 60 \]

   \[ F = 5 \times 6 \]

   \[ F = 30 \]

   \[ A = 3240 + 30 + 60 = 3630 \text{ in.}^2 \]

   Answer: 648 in.$^2$
Chapter 11-5 Volume of a Prism

PART I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. [12]

1. If each edge of a cube measures 3 units, then the volume of the cube is
   (1) 6 cu units
   (2) 9 cu units
   (3) 12 cu units
   (4) 27 cu units

2. What is the surface area of a cube with a volume of 64 cubic units?
   (1) 16$x^2$ sq units
   (2) 16$x^3$ sq units
   (3) 64$x^2$ sq units
   (4) 96$x^2$ sq units

3. If the surface area of a cube is 96 square inches, what is the volume of the cube?
   (1) 8 in.$^3$
   (2) 16 in.$^3$
   (3) 64 in.$^3$
   (4) 256 in.$^3$

4. If each edge of a rectangular solid is doubled, the volume is multiplied by
   (1) 2
   (2) 3
   (3) 4
   (4) 8

5. The rectangular prism has a volume of 1 cubic unit. If the dimensions of the prism are $n, \frac{1}{n},$ and $x$, what is the value of $x$?
   (1) 1
   (2) 2
   (3) $n$
   (4) $\frac{1}{n}$

6. A triangular prism has a base that is an equilateral triangle with sides of length 4. If one lateral edge of the prism has length 6, what is the volume of the prism?
   (1) $8\sqrt{3}$ cu units
   (2) 24 cu units
   (3) $24\sqrt{3}$ cu units
   (4) 48 cu units
PART II

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [8]

7. A cube with an edge measuring 2 inches is filled with water and its contents are emptied into a larger cube with an edge measuring 4 inches. The smaller cube is again filled with water, and the water is dumped into the larger cube. What is the height in inches of the water in the larger cube?

Answer: 1 inch

Solution: The volume of the cube is 8 cubic inches. Thus, the volume of the water emptied into the larger cube is 2(8) or 16 cubic inches. Since the dimensions of the base of the larger cube are 4 by 4 inches, the height of the water can be found by solving the following equation:

\[ h = \frac{16}{4^2} = \frac{16}{16} = 1 \text{ inch} \]

8. a. A right prism has a rhombus for a base. If the diagonals of the rhombus have lengths 10 and 14 and the volume of the prism is 455 cubic units, what is the height of the prism?

\[ V = Bh \]

\[ 455 = \frac{1}{2} \times 10 \times 14 \times h \]

\[ 455 = 70h \]

\[ h = \frac{455}{70} = 6.5 \text{ in.} \]

b. Find the volume of a cube with a lateral area of 100 square inches.

Let the side of the cube be \( s \). Then the lateral area of the cube is \( 4s^2 \). We are given that \( 4s^2 = 100 \) square inches.

\[ s^2 = \frac{100}{4} \]

\[ s^2 = 25 \]

\[ s = 5 \text{ in.} \]

The volume of the cube is \( s^3 \).

\[ V = 5^3 = 125 \text{ in.}^3 \]
Chapter 11-6 Pyramids

PART I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. [12]

1. Each lateral face of a regular pyramid is bounded by what type of triangle?
   (1) scalene triangle
   (2) right triangle
   (3) isosceles triangle
   (4) equilateral triangle

2. In the given figure, a pyramid fits exactly in the box.

What is the ratio of the volume of the pyramid to the volume of the box?
   (1) \( \frac{1}{4} \)
   (2) \( \frac{1}{3} \)
   (3) \( \frac{1}{2} \)
   (4) \( \frac{2}{3} \)

3. A regular square pyramid has a height of 12, a slant height of 13, and the edges of the base each have a length of 10. What is the lateral surface area of the pyramid?
   (1) 260 sq units
   (2) 312 sq units
   (3) 360 sq units
   (4) 400 sq units

4. What is the altitude of a square pyramid if its volume is 75 cubic feet and a side of the base measures 5 feet?
   (1) 3 ft
   (2) 6 ft
   (3) \( 8.3 \) ft
   (4) 9 ft

5. Find the volume of a pyramid if the altitude of the pyramid is 18 meters and the base is a right triangle with legs measuring 7 meters and 10 meters.
   (1) 126 m\(^3\)
   (2) 189 m\(^3\)
   (3) 210 m\(^3\)
   (4) 378 m\(^3\)

6. The sides of the base of a regular triangular pyramid each have a length of 8, a lateral edge has a length of 5, and the slant height is 3. Find the total surface area of the pyramid.
   (1) \( \frac{9\sqrt{7}}{4} + 24 \) sq units
   (2) \( \frac{9\sqrt{3}}{4} + 36 \) sq units
   (3) \( 16\sqrt{3} + 24 \) sq units
   (4) \( 16\sqrt{3} + 36 \) sq units
PART II

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [8]

7. a. A tetrahedron is a pyramid whose base and faces are congruent equilateral triangles. Find, to the nearest tenth of an inch, the surface area of a tetrahedron with edges measuring 6 inches. (Hint: The area of an equilateral triangle is \( s^2 \sqrt{3}/4 \) where \( s \) is the length of a side.)

\[ S = 4 \times \frac{s^2 \sqrt{3}}{4} = s^2 \sqrt{3} \]

\[ S = 6^2 \sqrt{3} \approx 240 \text{ in.}^2 \]

b. If the length of an edge of a regular tetrahedron is \( e \), what is the surface area of the tetrahedron expressed in terms of \( e \)?

\[ S = 4 \times \frac{e^2 \sqrt{3}}{4} = e^2 \sqrt{3} \]

8. a. Find, to the nearest tenth of a centimeter, the volume of a regular hexagonal pyramid with an altitude of 2 centimeters and a base with sides measuring 4 centimeters. (Hint: A hexagon is the union of six equilateral triangles.)

\[ V = \frac{1}{3} \times \text{base area} \times \text{height} \]

\[ V = \frac{1}{3} \times \frac{3 \times 4^2 \sqrt{3}}{4} \times 2 \]

\[ V = \frac{1}{3} \times 12 \sqrt{3} \times 2 \]

\[ V = 8 \sqrt{3} \approx 13.9 	ext{ cm}^3 \]

b. The length of a base edge of a square pyramid is \( e \) and the height of the pyramid is \( h \). What is the volume, \( V \), of the pyramid expressed in terms of \( e \) and \( h \)?

\[ V = \frac{1}{3} \times e^2 \times h \]
Chapter 11-7 Cylinders

PART I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. [12]

1. What is the volume of a cylinder with a diameter of 6 and an altitude 10?
   (1) $30\pi$ cu units
   (2) $90\pi$ cu units
   (3) $180\pi$ cu units
   (4) $360\pi$ cu units

2. A cylinder with a height of 5 inches has a volume of $45\pi$ square inches. What is the radius of the cylinder?
   (1) 3 in.
   (2) 4 in.
   (3) 5 in.
   (4) 9 in.

3. What is the total surface area of a right circular cylinder with a height of 5 inches and a radius of 4 inches?
   (1) $40\pi$ in.$^2$
   (2) $48\pi$ in.$^2$
   (3) $56\pi$ in.$^2$
   (4) $72\pi$ in.$^2$

4. A right circular cylinder has a base with a circumference of 12$\pi$. If the volume of the cylinder is $216\pi$ cubic units, what is the height of the cylinder?
   (1) 4
   (2) 6
   (3) 12
   (4) 16

5. In terms of $\pi$, what is the total surface area of a right circular cylinder with radius $r$ and height $2r$?
   (1) $2\pi r^2$ sq units
   (2) $4\pi r^2$ sq units
   (3) $4\pi r^3$ sq units
   (4) $6\pi r^2$ sq units

6. If the radius, $r$, of a right circular cylinder of height $h$ and volume $V$ is given by $r = \sqrt{\frac{V}{\pi h}}$, what is $h$ in terms of $r$ and $V$?
   (1) $\frac{\pi r^2}{V}$
   (2) $\frac{(\pi r)^2}{V}$
   (3) $\frac{V}{\pi r^2}$
   (4) $\frac{V}{\pi r}$
PART II

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [8]

7. The radius of a right circular cylinder is 3 inches and its volume is $96\pi$ cubic inches. Find, in terms of $\pi$:
   a. the height of the cylinder.
   
   b. the lateral surface area.
   
   c. the total surface area.

8. The height of a right circular cylinder is 2 and the diameter is 4. Which causes a greater change in the volume of the cylinder: doubling the diameter or doubling the height? Justify your answer.
1. What is the volume of a right circular cone with radius 6 and altitude 2?
   (1) $8\pi$ cu units
   (2) $24\pi$ cu units
   (3) $36\pi$ cu units
   (4) $72\pi$ cu units

2. If the slant height of a cone measures 10 and the radius of the base is 8, what is the volume of the cone?
   (1) $32\pi$ cu units
   (2) $120\pi$ cu units
   (3) $128\pi$ cu units
   (4) $192\pi$ cu units

3. The altitude of a cone is 15, the slant height is 17, and the radius is 8. What is the lateral area of the cone?
   (1) $68\pi$ sq units
   (2) $136\pi$ sq units
   (3) $480\pi$ sq units
   (4) $344\pi$ sq units

4. In the given figure, a cone fits exactly in the cylinder. What is the ratio of the volume of the cone to the volume of the cylinder?
   (1) $\frac{1}{3}$
   (2) $\frac{1}{4}$
   (3) $\frac{1}{5}$
   (4) $\frac{2}{3}$

5. The radius of the base of a cone measures 3 units and the altitude measures 7 units. What is the volume of the cone?
   (1) $21\pi$
   (2) $31.5\pi$
   (3) $49\pi$
   (4) $63\pi$

6. The volume of a given cone is three times the volume of a certain pyramid. If the bases have the same area, find the height, $x$, of the cone in terms of the height of the pyramid, $h$.
   (1) $\frac{h}{3\pi}$
   (2) $\frac{h}{3}$
   (3) $\frac{2h}{3}$
   (4) $3h$
PART II

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [8]

7. A right circular cone has an altitude of 12 inches and a slant height of 13 inches. Find, in terms of \( \pi \):
   a. the lateral surface area.
   b. the volume.

8. If the circumference of the base of a right circular cone is \( 9.6\pi \) meters and the height is 10 meters, what is the volume of the cone? (Leave your answer in terms of \( \pi \).)
1. What is the volume of a sphere with a 6 inch radius?
   (1) 72\pi \text{ in.}^3
   (2) 108\pi \text{ in.}^3
   (3) 144\pi \text{ in.}^3
   (4) 288\pi \text{ in.}^3

2. If the surface area of a sphere is 4 square units, then the radius of the sphere, to the nearest hundredth, is
   (1) 0.32
   (2) 0.56
   (3) 0.98
   (4) 1.77

3. The diameter of a sphere is 6 inches. What is the surface area of the sphere?
   (1) 12\pi \text{ in.}^2
   (2) 36\pi \text{ in.}^2
   (3) 108\pi \text{ in.}^2
   (4) 144\pi \text{ in.}^2

4. If the surface area of a sphere is equal to the surface area of a cube with an edge of length 4\pi, what is the radius of the sphere?
   (1) 2\sqrt{\pi}
   (2) 2\sqrt{6\pi}
   (3) 2\pi
   (4) 12\pi

5. If the volume of a sphere of radius \( r \) is equal to the volume of a right circular cylinder of radius 2\( r \), then what is the height of the cylinder in terms of \( r \)?
   (1) \( \frac{r}{4} \)
   (2) \( \frac{r}{3} \)
   (3) \( \frac{r}{2} \)
   (4) \( r \)

6. The base of a cone is congruent to a great circle of a sphere. If the circumference of the base of the cone is \( \pi \), what is the volume of the sphere?
   (1) \( \frac{1}{5}\pi \) cu units
   (2) \( \frac{1}{2}\pi \) cu units
   (3) \( \frac{2}{3}\pi \) cu units
   (4) 1.5\( \pi \) cu units
PART II

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [8]

7. The surface area of a sphere is $36\pi$ square meters. Find, to the nearest tenth of a meter:
   a. the radius.
   
   b. the volume.

8. The volume of the sphere is $\frac{32\pi}{3}$ square units. Find, in terms of $\pi$:
   a. the radius.
   
   b. the surface area.
Chapter 11 The Geometry of Three Dimensions

PART I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. [16]

1. A pyramid must have at least how many lateral faces?
   (1) 3
   (2) 4
   (3) 5
   (4) 6

2. What is the volume of a cube with a surface area of 600 square inches?
   (1) 60 in.³
   (2) 100 in.³
   (3) 900 in.³
   (4) 1,000 in.³

3. Which of the following is a point in the plane determined by \( \overrightarrow{BD} \) and \( C \)?
   (1) \( A \)
   (2) \( E \)
   (3) \( F \)
   (4) \( G \)

4. If the length of an edge of the base of a regular triangular pyramid is 6 and the slant height is 4, what is the lateral area?
   (1) 18 sq units
   (2) 24 sq units
   (3) 36 sq units
   (4) 48 sq units

5. In a regular pyramid, any two lateral edges are:
   (1) congruent
   (2) congruent and parallel
   (3) parallel only
   (4) perpendicular only

6. What is the volume of a rectangular prism with dimensions 2 by 3 by 4?
   (1) \( 8\sqrt{3} \) cu units
   (2) 18 cu units
   (3) 24 cu units
   (4) \( 24\sqrt{3} \) cu units

7. The circumference of the base of a right circular cylinder is \( 10\pi \). If the volume of the cylinder is \( 120\pi \) cubic units, then the height is
   (1) 2.4
   (2) 4.8
   (3) 5
   (4) 12

8. In the given cube, a triangle is to be drawn with \( F \) and \( E \) as two of the vertices.
   Which of the following points should be the third vertex if the triangle is going to have the largest possible perimeter?
   (1) \( A \)
   (2) \( C \)
   (3) \( D \)
   (4) \( G \)
PART II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [4]

9. A cone of radius 5 inches has a lateral area of $40\pi$ square inches. Find the slant height of the cone.

10. Two right circular cylinders $A$ and $B$, with radii of 3 and 5, respectively, have equal volumes. If the height of cylinder $A$ is 10, what is the height of cylinder $B$?

PART III

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [8]

11. What is the surface area of a sphere whose volume is $\frac{500\pi}{3}$ cubic units?
12. In the figure on the right, a sphere is inscribed in a cylinder. If the radius of the sphere is \( r \), what is the ratio of the volume of the sphere to the volume of the cylinder?

\[ \frac{\text{Volume of sphere}}{\text{Volume of cylinder}} = \frac{2}{3} \]

Solution:
Since the sphere is inscribed in the cylinder, the height of the cylinder is equal to the diameter of the sphere or \( 2r \) and the radius of the base of the cylinder is \( r \). Thus, the volume of the cylinder is \( \pi r^2 (2r) = 2\pi r^3 \) cubic units. The ratio of the volumes is then:

\[ \frac{\text{Volume of sphere}}{\text{Volume of cylinder}} = \frac{\frac{4}{3}\pi r^3}{2\pi r^3} = \frac{2}{3} \]

PART IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

13. a. A solid metal cylinder with a radius of 3 centimeters and a height of 10 centimeters is melted down and recast as a cone with a radius of 6 centimeters. Find the height of the cone.

Answer: 7.5 cm

Solution:
The volume of the cylinder is \( \pi (3^2) (10) = 90\pi \) cubic centimeters. The cone will have the same volume. Thus:

\[ \frac{1}{3} \pi r^2 h = 90\pi \]

b. If the volume of cone \( A \) equals the volume of sphere \( B \), and the radius, \( r \), of the cone equals the radius, \( r \), of the sphere, find the height, \( h \), of the cone in terms of \( r \).
14. In the right circular cone shown to the right, the altitude is 4 and the radius is 3. Find, in terms of $\pi$:

a. the slant height.

b. the lateral area.

c. the total area.

d. the volume.
1. Which statement is always true?
   (1) A rhombus is a square.
   (2) A rectangle is a parallelogram.
   (3) A parallelogram is a square.
   (4) A parallelogram is a rectangle.

2. If \( 6 - x > 0 \) and \( 5x + 2 > -8 \), then \( x \) could equal each of the following except
   (1) \(-2\)  (3) \(2\)
   (2) \(0\)  (4) \(4\)

3. If each interior angle of an equiangular polygon is a 144\(^\circ\) angle, how many sides does this polygon have?
   (1) \(5\)  (3) \(10\)
   (2) \(8\)  (4) \(12\)

4. Which of the following is the equation of the line that passes through the point \((1, 5)\) and is parallel to the \(x\)-axis?
   (1) \(x = 1\)
   (2) \(x = 5\)
   (3) \(y = 1\)
   (4) \(y = 5\)

5. If the measure of an angle is \(2x - 10\), what is the measure of its complement?
   (1) \(80 - 2x\)
   (2) \(100 - 2x\)
   (3) \(2x - 100\)
   (4) \(190 - 2x\)

6. Under which operation is the set of odd integers closed?
   (1) addition
   (2) subtraction
   (3) multiplication
   (4) division

7. In the given figure, \(\overline{AB} \equiv \overline{BD}\) and points \(A, D,\) and \(C\) are collinear.

8. Given: \(f \rightarrow g\) and \(h \rightarrow \sim g\)
   Which statement follows from the premises?
   (1) \(f \rightarrow h\)
   (2) \(f \rightarrow \sim h\)
   (3) \(g \rightarrow h\)
   (4) \(\sim g \rightarrow f\)

Copyright © Amsco School Publications, Inc.
9. Two parallel lines are cut by a transversal. If the measures of two interior angles on the same side of the transversal are $x$ and $3x - 40$, find the measure of the larger angle.

Answer: 125°

Solution: The interior angles on the same side are supplementary. Thus: $x + 3x - 40 = 180°$. Therefore, the measure of the larger angle is $3x - 40 = 125°$.

10. The length of the diagonal of a square is 8 inches. Find the area of the square.

Answer: 32 in.²

Solution: The diagonal and the sides of the square form a right triangle. Thus, if $x$ represents the length of a side of the square:

$$x^2 + x^2 = 8^2$$

$$2x^2 = 64$$

$$x^2 = 32$$

Therefore, the area of the square is $32$ in.².
PART III

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [8]

11. If the equations of two parallel lines are $4x + ay = 12$ and $x + 6y = 12$, what is the value of $a$?

12. A solid block of aluminum is in the shape of a rectangular solid. The dimensions are 2 by 2 by 16 centimeters. If the aluminum is melted down and recast as a cube, what is the difference in the surface area between the rectangular solid and the cube?
PART IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

13. The coordinates of the vertices of \( \triangle PET \) are \( P(6, 2) \), \( E(-4, 4) \), and \( T(-2, -4) \).

   a. Find the coordinates of the midpoint \( M \) of side \( \overline{PE} \).

   b. Find the coordinates of the midpoint \( K \) of side \( \overline{ET} \).

   c. Using coordinate geometry, show that \( MK \parallel PT \).
14. Given: $\triangle PQR$, $\overline{AC} \equiv \overline{CB}$, and $\angle 1 \equiv \angle 2$.

Prove: $\angle P \equiv \angle R$