The postulates and theorems in this section use the word “determine.” Be sure students understand that a figure is determined when there is one and only one such figure that meets a set of given conditions.

Exercise 1 calls attention to the fact that an alternate definition of skew lines is lines that do not lie in the same plane. Students should explain that, by definition, parallel lines are coplanar and, by Theorem 11.2, intersecting lines are coplanar. So lines that do not lie in the same plane are neither intersecting nor parallel; thus, these lines are skew.

For Exercise 13, note that the illustration shows an arrangement of four points that determine four planes (ABC, ADC, BCD, and ADB). Students may recall that the number of combinations of four points taken three at a time is \( \binom{4}{3} = 4 \).

**II-2 PERPENDICULAR LINES AND PLANES**

The half-planes that form the dihedral angle are sometimes called the faces of the dihedral angle. A dihedral angle is called acute, right, obtuse, or straight if its plane angle is acute, right, obtuse, or straight. The point of intersection of a line and a plane is sometimes called the foot of the line.

From the discussion in this section, students may identify ways that planes act in three-dimensional space much as lines do in...
Students should know and be able to apply the results of Theorems 11.3 and 11.9. They should also recognize general approaches that are useful when they are asked for a proof. For example, to prove the measures of two line segments are equal, they should identify two triangles containing these segments and prove the triangles are congruent; to prove that a given line is perpendicular to a given plane, they should show (or explain that it is necessary to show) that the given line is perpendicular to two intersecting lines in the given plane.

The emphasis for the results in this section is on applications and understanding, as shown by the exercises. A typical application of Theorem 11.4 would be to present students with a drawing of a pyramid whose base is quadrilateral $ABCD$ and ask students to explain what must be done to prove that $EF$ is the altitude of the pyramid. (Show that $EF \perp AC$ and $EF \perp DB$.)

An application of Theorem 11.5 would be to describe a situation where a contractor wants to build a wall in the attic of a house. Then ask students to explain what the contractor must do to ensure that the wall is perpendicular to the attic floor and to give the theorem that justifies their answer.

11-3 PARALLEL LINES AND PLANES

As in the previous section, several of the results for parallelism in three dimensions are similar to results for two dimensions. Again, students should be expected to know and apply the results of Theorem 11.10 to 11.13, and may be asked to explain the indirect proof of Theorem 11.10.

Theorem 11.11 states that two lines perpendicular to the same plane are parallel. Ask students if, in three-dimensional space, two lines perpendicular to the same line are necessarily parallel. The answer is no. As the figure below shows, $AB$ is perpendicular to $BC$, $DC$ is perpendicular to $BC$, but $AB$ and $DC$ are not parallel.

Algebraic applications of Theorem 11.13 are easily constructed. In the figure below, plane $m$ is parallel to plane $n$, and $AB$ and $CD$ are perpendicular to both planes. If $AB$ is represented by $3x - 1$ and $CD$ is represented by $x + 9$, what is the distance between the two planes? (14 units)

11-4 SURFACE AREA OF A PRISM

If possible, have students use centimeter graph paper for the manipulative activities in the section and the exercises, so that figures constructed are easier to handle.

A prism whose lateral edges are not perpendicular to the bases (that is, not a right
A more formal proof of this result involves *Cavalieri's Principle*, which says that if two three-dimensional solids are contained between two parallel planes such that every plane parallel to the two given planes cuts cross-sections of the solids with equal areas, then the volumes of the solids are equal.

From the general formula for the volume of a prism, it follows that the volume of a rectangular solid with dimensions $l$, $w$, and $h$ is $V = lwh$, and the volume of a cube whose edge length is $s$ is $V = s^3$.

### 11-6 PYRAMIDS

Emphasize that prisms and pyramids differ in the shape of their lateral faces: the lateral faces of right prisms are rectangles and the lateral faces of regular pyramids are isosceles triangles. The base(s) of either solid can be any polygon.

As shown in the example, when we are working with pyramids, it is a good idea to differentiate the height of the pyramid $h_p$ and the slant height of each triangular face $h_s$.

The figure below can be used to justify the volume formula for a square pyramid. Start with a cube and consider the four diagonals from a specific vertex to the other vertices. Taking the diagonals three at a time, we can recognize three pyramids inside the cube. The pyramids are congruent and intersect only in faces or edges, so each pyramid fills one-third of the cube. Hence, the volume of each pyramid is one-third of the volume of the cube.
11-7 CYLINDERS

If we visualize a series of prisms with increasing numbers of sides in their bases, a cylinder can then be considered the limiting figure for the process. Since the volume of each prism is the product of the area of its base and its height, we would expect the same to be true for a cylinder. This expectation can be proved using calculus.

The formula for the volume of a cylinder is valid for oblique as well as right cylinders. In fact, for cylinders whose bases are other simple closed curves, the volume is still the product of the area of the base and the height.

11-8 CONES

We can determine the volume of a cone using a procedure similar to the one for cylinders; that is, consider a sequence of pyramids with increasing numbers of side in the bases. Since the volume of each pyramid is one-third of the volume of the smallest prism containing it, we can reasonably expect the volume of the cone to be one-third the volume of the smallest cylinder containing it. Again, the proof can be done using calculus.

For a right circular cone, the slant height is the distance from the vertex (or apex) of the cone to the base of the cone measured along the surface of the cone. If the height of the cone is \( h_c \) and the radius of the base is \( r \), then the slant height is \( h_s = \sqrt{h_c^2 + r^2} \) (by the Pythagorean Theorem) and the lateral area is \( \pi r \sqrt{h_c^2 + r^2} \). Adding the area of the circular base gives the total surface area \( \pi r^2 + \pi r \sqrt{h_c^2 + r^2} \).

11-9 SPHERES

The Greek mathematician Archimedes (287–212 B.C.) discovered and verified formulas for the surface area and volume of a sphere. His method for deriving the volume of a sphere, called the Archimedean method, applied the principle of the lever. He compared a sphere of radius \( r \), a cone of radius \( 2r \) and height \( 2r \), and a cylinder of radius \( 2r \) and height \( 2r \). Using cross-sections, Archimedes concluded that a solid cone plus a solid sphere at a distance of 2 units from the lever’s fulcrum would balance the solid cylinder at a distance of 1 unit from the fulcrum.

This implies that the combined volume of the cone and sphere equals one-half the volume of the cylinder. Since it was known at that time that the cone’s volume was one-third that of the cylinder, it followed that the sphere’s volume must be one-sixth the cylinder’s volume, so the volume of the sphere is

\[
\frac{1}{6}(8\pi r^3) = \frac{4}{3}\pi r^3.
\]

Archimedes believed his approach was useful for providing insight into mathematical relationships, but that it did not constitute a rigorous proof. However, Archimedes was able to apply alternate mathematical reasoning to justify his conclusion and mathematicians view his work to be as rigorous as a calculus-based approach.

Students should know the correct terminology and properties of the sphere and be able to describe relationships among cylinders, cones, and spheres. For example, given a cylinder, cone, and sphere with the same radius and same height for the cylinder and cone, what is the relationship among the three volumes? Which is greatest? If the radius of a sphere is doubled, how is its surface area changed? How is its volume changed?

Explain that if Earth were a perfect sphere, the equator and the circles formed by the meridians would be great circles. There are infinitely many great circles of a sphere.

**Enrichment Activity 11-9A: Volume Formulas** presents a general volume formula that can be used to derive the formulas given for cubes, spheres, pyramids, cones, cylinders, and prisms.

In **Enrichment Activity 11-9B: Geodesic Domes**, students will construct geodesic...
domes out paper clips and straws and estimate their surface area.

**EXTENDED TASK**

*For the Teacher:*

The five regular polyhedra are reviewed and students are introduced to another group of special solids called the semiregular polyhedra. To find information about these figures, students can consult books dealing with math history and solid geometry, or they may use the Internet. Patterns for paper models and directions for straw models are also available on the Internet.

To figure out the number of corners and edges each figure has, students may reason as follows. A truncated octahedron has 14 faces, of which 6 are squares and 8 are hexagons. A square has 4 corners, so 6 squares have a total of 24 corners. A hexagon has 6 corners, so 8 hexagons have 48 corners. Adding squares and hexagons gives 72 corners. Since three faces meet at each corner of the solid, the solid has $72 \div 3 = 24$ corners in all. A polygon always has the same number of sides as it has corners, so the figure has 72 sides. Since two sides meet at each edge, the solid has $72 \div 2 = 36$ edges in all.

As a class, have students use the following information and the reasoning above to find the number of corners and the number of edges in each figure.

**Truncated icosahedron:**
- 32 faces of which 12 are pentagons and 20 are hexagons;
- 2 hexagons and 1 pentagon meet at each corner.

**Great rhombicuboctahedron:**
- 26 faces of which 12 are squares, 8 are hexagons, and 6 are octagons;
- 1 square, 1 hexagon, and 1 octagon meet at each corner.

**Small rhombicosidodecahedron:**
- 62 faces of which 20 are triangles, 30 are squares, and 12 are pentagons;
- 2 squares, 1 triangle, 1 pentagon meet at each corner.

**Snub dodecahedron:**
- 92 faces of which 80 are triangles and 12 are pentagons;
- 4 triangles and 1 pentagon meet at each corner.

Note that some of the solids may appear under different names. For example, the *great rhombicuboctahedron* is also called the *truncated cuboctahedron.*
**Volume Formulas**

A polyhedron in which the bases are polygons in parallel planes is called a **prismatoid**. Unlike a prism, the bases of a prismatoid are not necessarily congruent polygons. (All prisms are prismatoids but not all prismatoids are prisms.)

The **prismoidal formula** below is a general expression for finding the volume of any prismatoid:

\[
V = \frac{h}{6} (B_1 + 4M + B_2)
\]

\(B_1\) and \(B_2\) represent the area measures of the bases and \(h\) represents the distance between the bases, that is, the height of the figure. The plane section halfway between and parallel to the bases is the **midsection**, and \(M\) represents the measure of the area of the midsection.

1. For a cube with sides of length \(s\), \(B_1 = B_2 = M = s^2\), and \(h = s\). Show that the prismoidal formula gives the standard expression for the volume.

2. The prismoidal formula can be used to derive an expression for the volume of a sphere of radius \(r\). Study the figure below and think of the “bases” of the sphere as the points of tangency in each of two parallel planes.

   - **a.** What are \(B_1\) and \(B_2\)?
   - **b.** What is \(M\)?
   - **c.** What is \(h\)?
   - **d.** Use the prismoidal formula to find an expression for the volume of the sphere.
3. For a square pyramid with height $h$ and base with edge $s$, the midsection is a square with edges of length $\frac{1}{2}s$. Use the prismoidal formula to find an expression for the volume of a square pyramid. (*Hint:* What is $B_2$?)

4. For a right circular cone with height $h$ and radius $r$, the midsection is a circle with radius $\frac{1}{2}r$. Use the prismoidal formula to find an expression for the volume of a right circular cone. (*Hint:* What is $B_2$?)

5. Show that the prismoidal formula gives the standard volume formula for a right circular cylinder of radius $r$ and height $h$. 
Geodesic Domes

Geodesic domes are spherically shaped structures that get their stability from their triangular faces and their efficiency from the characteristics of the sphere. These domes take their form from the icosahedron, a regular solid having 30 equal edges, 12 vertices, and 20 equilateral triangles as faces. (See Chapter 11, Exploration.)

You can construct a geodesic dome using paper clips and straws. (The paper clip has to fit snugly into the end of the straw so be sure to check the width of each item.) Cut 30 equal struts, 3-inch to 4-inch straw lengths, and assemble them as shown.

**Step 1**
Slip paper clips onto one clip for connecting corners.

**Step 2**
Connect 5 triangles that form the 5 sides. Slide paper clip B into straws B and paper clips A into straws A.

**Step 3**
Fold triangles around so that they form a 5-sided structure. Notice pentagon-shaped base.

**Step 4**
This structure will form the top of the dome. Repeat this step to form the bottom of the dome.

**Step 5**
Connect top and bottom of dome.

Slip 5 straws onto the 5 connected paper clips.
The dome you constructed is called a one-frequency dome. To make a more spherical and stable structure, each face of the icosahedron can be divided into smaller congruent triangles by dividing the edge by 2, 3, 4, and so on.

1. Find a pattern that can be used to help you determine the number of smaller triangles found in an \( n \)-frequency dome.

2. The shape of a dome can also be changed by using a combination of equilateral and isosceles triangles with different strut lengths. Use library resources and the Internet to research two-frequency domes and find instructions for constructing one.

3. Estimate the surface area of the dome you constructed by finding the sum of the areas of the triangles. The area of an equilateral triangle is \( \frac{\sqrt{3}}{4} s^2 \) where \( s \) is the length of a side.

4. a. Estimate the volume of your dome. Think of the top and bottom as pyramids, and estimate the central section as a prism.
   b. Estimate the surface area of a rectangular prism with the same volume.
   c. Which structure uses more material (surface area) for the same amount of space (volume)?
Archimedean Solids

There are only five regular polyhedra or Platonic solids, solids with faces in the shape of regular polygons of the same type and with the same number of polygons at each corner or vertex. These regular polyhedra are the cube (3 squares at each vertex, 6 faces); the tetrahedron (3 equilateral triangles at each vertex, 4 faces); the octahedron (4 triangles, 8 faces); the icosahedron (5 triangles, 20 faces); and the dodecahedron (3 regular pentagons; 12 faces).

Another set of geometric solids are the semiregular polyhedra. These are solids with faces that are all regular polygons and with corners that are all the same. Unlike regular polyhedra, the faces of a semiregular polyhedron are not necessarily congruent. (All regular polyhedra are semiregular but not all semiregular polyhedra are regular polyhedra.)

Of particular interest among the semiregular polyhedra are thirteen solids called the Archimedean solids. The names of these solids are listed in the chart below. For example, a truncated tetrahedron has 8 faces of which 4 are triangles and 4 are hexagons. At each vertex, two hexagons and one triangle meet. We say that the vertex arrangement is 3-6-6. The 3 followed by the two 6’s tells us that a triangle and two hexagons meet at a vertex. A truncated tetrahedron has 12 vertices and 18 edges.

a. Complete the chart. The type and number of polygons for each Archimedean solid has been given along with the vertex arrangement. Use this information to determine the number of faces, edges, and vertices (corners) for each of the thirteen solids.

b. Describe any patterns you observe in the chart. What patterns (if any) do you observe among the number of faces, edges, and vertices?

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>No. of Faces</th>
<th>No. of Edges</th>
<th>No. of Vertices</th>
<th>Vertex Arrangement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Truncated Tetrahedron</td>
<td>4 triangles, 4 hexagons</td>
<td>8</td>
<td>18</td>
<td>12</td>
<td>3-6-6</td>
</tr>
<tr>
<td>2. Cuboctahedron</td>
<td>8 triangles, 6 squares</td>
<td></td>
<td></td>
<td></td>
<td>3-4-3-4</td>
</tr>
<tr>
<td>3. Truncated Cube</td>
<td>8 triangles, 6 octagons</td>
<td></td>
<td></td>
<td></td>
<td>3-8-8</td>
</tr>
<tr>
<td>4. Truncated Octahedron</td>
<td>6 squares, 8 hexagons</td>
<td></td>
<td></td>
<td></td>
<td>4-6-6</td>
</tr>
<tr>
<td>5. Small Rhombicuboctahedron</td>
<td>8 triangles, 18 squares</td>
<td></td>
<td></td>
<td></td>
<td>3-4-4-4</td>
</tr>
<tr>
<td>6. Great Rhombicuboctahedron</td>
<td>12 squares, 8 hexagons, 6 octagons</td>
<td></td>
<td></td>
<td></td>
<td>4-6-8</td>
</tr>
<tr>
<td>Name</td>
<td>Description</td>
<td>No. of Faces</td>
<td>No. of Edges</td>
<td>No. of Vertices</td>
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<tr>
<td>7. Icosidodecahedron</td>
<td>20 triangles, 12 pentagons</td>
<td></td>
<td></td>
<td></td>
<td>3-5-3-5</td>
</tr>
<tr>
<td>8. Truncated Dodecahedron</td>
<td>20 triangles, 12 decagons</td>
<td></td>
<td></td>
<td></td>
<td>3-10-10</td>
</tr>
<tr>
<td>9. Truncated Icosahedron</td>
<td>12 pentagons, 20 hexagons</td>
<td></td>
<td></td>
<td></td>
<td>5-6-6</td>
</tr>
<tr>
<td>10. Snub Cube</td>
<td>32 triangles, 6 squares</td>
<td></td>
<td></td>
<td></td>
<td>3-3-3-3-4</td>
</tr>
<tr>
<td>11. Small Rhombicosidodecahedron</td>
<td>20 triangles, 30 squares, 12 pentagons</td>
<td></td>
<td></td>
<td></td>
<td>3-4-5-4</td>
</tr>
<tr>
<td>12. Great Rhombicosidodecahedron</td>
<td>30 squares, 20 hexagons, 12 decagons</td>
<td></td>
<td></td>
<td></td>
<td>4-6-10</td>
</tr>
<tr>
<td>13. Snub Dodecahedron</td>
<td>80 triangles, 12 pentagons</td>
<td></td>
<td></td>
<td></td>
<td>3-3-3-3-5</td>
</tr>
</tbody>
</table>
Geometry: Chapter Eleven Test

PART I
Write your answers legibly in the space provided below. Show any work on scratch paper. An incorrect answer with sufficient work may receive partial credit. A correct answer with insufficient work may only receive partial credit. All scratch paper must be turned in at the conclusion of this test.

In 1–8, answer each question and state the postulate, theorem, or definition that justifies your answer.

1. Points A, B, and C are three non-collinear points. Is there a plane that contains line \( \overrightarrow{AC} \) and also point B?

2. Point M is on plane \( p \). \( \overrightarrow{KM} \) and \( \overrightarrow{LM} \) are two distinct lines that intersect at point M. Is it possible for both \( \overrightarrow{KM} \) and \( \overrightarrow{LM} \) to be perpendicular to \( p \) at M?

3. Lines \( \overrightarrow{AB} \) and \( \overrightarrow{CD} \) do not intersect. Must \( \overrightarrow{AB} \) be parallel to \( \overrightarrow{CD} \)?

4. Plane g is parallel to plane f and plane k is parallel to plane g. Is plane f parallel to plane k?

5. \( \overrightarrow{MN} \perp p \) and \( \overrightarrow{RS} \perp p \). Is there a plane \( q \) that contains \( \overrightarrow{MN} \) and \( \overrightarrow{RS} \)?

6. Can the base of a regular pyramid be a rhombus?

7. X and Y are two points on a sphere which do not form a diameter. Does the sphere have a great circle that passes through X and Y?

8. Can a prism be constructed with exactly four faces?
In 9–13, find the exact lateral area, total surface area, and volume of each solid figure.

9. The length of each side of the square base of a rectangular prism is 7 centimeters and its height is 13 centimeters.

10. The height of a prism with bases that are right triangles is 6 inches. The lengths of the sides of the bases are 10, 24, and 26 inches.

11. The radius of a cone is 6 centimeters, its height is 8 centimeters, and its slant height is 10 centimeters.

12. The diameter of the base of a right circular cylinder is 42 millimeters and its height is 60 millimeters.

13. A pyramid has a square base with an edge that measures 14 inches. The slant height of a lateral side is 25 inches and the height of the pyramid is 24 inches.

14. Find the volume and surface area, to the nearest whole unit, of a sphere whose diameter is 24 centimeters.

15. In two right circular cones, the altitudes are congruent and the ratio of the measures of the radii of their bases is 3 : 1. Find the ratio of the volumes of the two cones.

16. The volume of a cube is equal to the volume of a rectangular solid that has a length of 20 inches, a width of 5 inches, and a height of 10 inches. Find the perimeter of one face of the cube.

17. Over a rectangular driveway 120 feet long and 9 feet wide, a layer of gravel is to be spread to an average depth of 4 inches. Find the number of cubic feet of gravel that must be used.

18. The volume of a sphere is $36\pi$ cubic meters. Find, in terms of $\pi$, the surface area of the sphere.

19. Find the volume and surface area of the trapezoidal prism shown.
20. The volume of a rectangular solid is 1,430 cubic meters. The width is 1 meter less than the length and the height is 2 meters more than the length.
   a. Find the dimensions of the solid.
   b. Find the surface area of the solid.

PART II
Write your answers legibly on a separate piece of paper. Show any work. An incorrect answer with sufficient work may receive partial credit. A correct answer with insufficient work may only receive partial credit. All scratch paper must be turned in at the conclusion of this test.

21. Given: \( \overrightarrow{MA} \perp p, \overrightarrow{BM} \cong \overrightarrow{CM} \cong \overrightarrow{DM} \cong \overrightarrow{EM}, \)
    and \( \angle BMC \) is a right angle.
    Prove: The pyramid with vertex \( A \) and base \( BCDE \) is regular.

22. Given: Planes \( p, q, \) and \( r \) cut by plane \( s \) in \( AB, CD, \) and \( EF; p \parallel q \) and \( r \parallel q. \)
    Prove: \( AB \parallel EF \)

Bonus: Arrange the volumes of the following figures from smallest to largest.
1. A cube with edge length 10
2. A sphere with diameter 10
3. A right circular cone with diameter 10 and height 10
4. A right circular cylinder with diameter 10 and height 5
SAT Preparation Exercises (Chapter 11)

1. MULTIPLE-CHOICE QUESTIONS

In 1–12, select the letter of the correct answer.

1. In space, how many lines can be drawn perpendicular to a given line at a given point on the line?
   (A) none (B) 1 (C) 2 (D) 4 (E) an infinite number

2. How many edges does a right triangular prism have?
   (A) 4 (B) 5 (C) 6 (D) 9 (E) 12

3. If $\overrightarrow{AB}$ intersects plane $p$, how many planes containing $\overrightarrow{AB}$ can be drawn parallel to plane $p$?
   (A) none (B) 1 (C) 2 (D) 4 (E) an infinite number

4. If the volume of a cube is 8 cubic units, what is the shortest distance from the center of the cube to the base of the cube?
   (A) 1 (B) 2 (C) $\sqrt{2}$ (D) $\sqrt{2}$ (E) $2\sqrt{2}$

5. A right circular cylinder has a radius of 2 and height of 5. Point $X$ lies on the circumference of the top and point $Z$ lies on the circumference of the bottom. What is the greatest possible straight-line distance between $X$ and $Z$?
   (A) 3 (B) 5 (C) $\sqrt{21}$ (D) $\sqrt{29}$ (E) $\sqrt{41}$

6. The length of a rectangular solid is 6, the width is 2 more than the length, and the height is half the width. What is the volume of the solid?
   (A) 18 cubic units (B) 144 cubic units (C) 192 cubic units (D) 208 cubic units (E) 384 cubic units

7. The diameter of Cylinder I is 5 and its height is 9. The diameter of Cylinder II is 10 and its height is 9. What is the ratio of the volume of Cylinder I to Cylinder II?
   (A) 1 : 2 (B) 1 : 4 (C) 1 : 5 (D) 1 : 8 (E) 1 : 25

8. What is the surface area of a cube whose volume is 64?
   (A) 16 square units (B) 64 square units (C) 96 square units (D) 128 square units (E) 384 square units

9. Janet and Felix each roll a sheet of $8\frac{1}{2} \times 11$ paper to form a cylinder. Janet tapes the two $8\frac{1}{2}$-inch edges together. Felix tapes the two 11-inch edges together. Which of the following is true?
   (A) The volume of Janet’s cylinder is greater.
   (B) The volume of Felix’s cylinder is greater.
   (C) Both cylinders have the same volume.
   (D) Both cylinders have the same surface area.
   (E) The answer cannot be determined.

10. The height $h$ of a cylinder is equal to the edge of a cube. If the cylinder and the cube have the same volume, which expression represents the radius of the cylinder?
    (A) $h\sqrt{\pi}$ (B) $\frac{h}{\sqrt{\pi}}$ (C) $\frac{\sqrt{\pi}}{h}$
    (D) $\pi h^2$ (E) $\frac{\pi}{h^2}$

11. The volume of rectangular solid with a square base is 216 cubic inches. What is the least possible surface area for the solid?
    (A) 24 in.$^2$ (B) 72 in.$^2$ (C) 216 in.$^2$ (D) 252 in.$^2$ (E) 492 in.$^2$
12. A wooden cube whose edges are 4 inches is painted green. The cube is then cut into 64 one-inch cubes. How many small cubes have exactly 1 green face?
   (A) 4  (B) 8  (C) 12  (D) 16  (E) 24

II. STUDENT-PRODUCED RESPONSE QUESTIONS
In 13–20, you are to solve the problems.

13. The volume of a cone is 261 cubic inches. What is the volume of a right circular cylinder of the same height and with the same radius of the base?

14. Cube I has an edge of 4 units. Each edge of cube I is increased by 50%, creating a new cube, Cube II. How much greater, in square units, is the surface area of Cube II than Cube I?

15. Two right circular cylinders, I and II, have diameters of 12 and 18, respectively. If the volume of Cylinder II is twice the volume of Cylinder I, what is the ratio of the height of I to the height of II?

17. All the dimensions of a certain rectangular solid are integers greater than 1. If the volume of the solid is 168 cubic inches and the height is 8 inches, what is the perimeter of the base?

18. A roll of paper towels is 12 inches long and 6 inches in diameter. To the nearest whole unit, what is the area of plastic needed to shrink-wrap the roll?

19. The side of the base of a square pyramid is 5 feet and the pyramid has a slant height of 8 feet. The pyramid is to be completely covered in gold foil and the foil comes in rolls each containing 10 square feet of foil. What is the minimum number of rolls needed to cover the pyramid?

20. An open box will be made from a piece of cardboard 30 inches wide by 36 inches long by folding congruent square tabs at each corner. If 3-inch squares are cut at each corner and the sides are folded up, what is the volume in cubic feet of the box?